Improving a Linearly Implicit Quantized State System Method

Winter Simulation Conference 2016

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CIFASIS–CONICET

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december 2016

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Ordinary Differential Equations (ODEs) Models

Lumped parameter models coming from mechanics, electromagnetism, chemistry, thermodynamics, hydraulics, etc. are usually represented by sets of Ordinary Differential Equations of the form:

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1(t), \cdots, x_n(t), t) \\ \dot{x}_2(t) &= f_2(x_1(t), \cdots, x_n(t), t) \\ &\vdots \\ \dot{x}_n(t) &= f_n(x_1(t), \cdots, x_n(t), t) \end{aligned} \tag{1}$$

where t represents time, $x_i(t)$ are the state variables and $\dot{x}_i(t)$ represents $x_i(t)$ first time derivative.

Ordinary Differential Equations (ODEs) Models

The system represented by Eq.(1) can be written in a compact manner by using vector notation:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t) \tag{2}$$

where

$$\mathbf{x}(t) \triangleq [x_1(t), x_2(t), \cdots, x_n(t)]^T$$

is the states vector, for which initial conditions are usually known

$$\mathbf{x}(t_0) = \mathbf{x}_0 \tag{3}$$

Continuous Systems Simulation

In order to simulate system of Eq.(2), the equation must be solved from the initial condition $\mathbf{x}_0$.

In general, analytically solving Eq.(2) is impossible.

For that reason, Numerical Integration Methods of ODEs are used, which attempt to provide an approximated solution.

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Classic Numerical Integration Methods

Given the system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t)$$

with initial conditions $\mathbf{x}(t_0) = \mathbf{x}_0$, the aim of numerical integration methods is to obtain an approximated solution for times $t_1, t_2, \cdots, t_N$.

$$\tilde{\mathbf{x}}_1 \approx \mathbf{x}(t_1),\ \tilde{\mathbf{x}}_2 \approx \mathbf{x}(t_2), \cdots, \tilde{\mathbf{x}}_N \approx \mathbf{x}(t_N),$$

The difference $h_k \triangleq t_{k+1} - t_k$ is called integration step and may be constant or variable.

Classic Numerical Integration Methods

Single-step methods

These methods compute $\mathbf{x}_{k+1}$ just by using information about $\mathbf{x}_k$. (Runge–Kutta methods)

Multi-step methods

These methods compute $\mathbf{x}_{k+1}$ by using information about $\mathbf{x}_k$ and some other previous instants ($\mathbf{x}_{k-1}$, etc).

Implicit methods

Implicit methods (single of multi-step) use future information to compute $\mathbf{x}_{k+1}$, so an equation is required to be solved in every step.

- they present advantages as regards to numerical stability
- their implementation require iterative algorithms

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Stiff Systems

These systems simultaneously present slow and fast dynamics.

At first, a small step should be used, and then enlarge it as fast dynamics fades away.

Problem: explicit methods became numerically unstable when step h is enlarged.

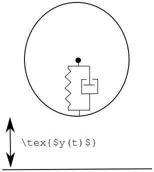
Hence, when dealing with stiff systems, the use of implicit algorithms with step control is mandatory.

Discontinuous Systems

The model of a simple ball falling and bouncing on the floor is the following:

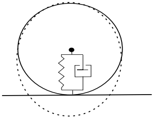
$$\dot{y}(t) = v(t)$$

$$\dot{v}(t) = \begin{cases} -g & \text{if } y(t) > 0 \\ -g - \frac{k}{m} \cdot y(t) - \frac{b}{m} \cdot v(t) & \text{if } y(t) \leq 0 \end{cases}$$



This ODE has a discontinuity in $y = 0$.

Integration methods might produce unacceptable errors. Detection of instants in which $y(t) = 0$ in necessary, and from that point, the simulation must restart.



Some problematic cases

There are diverse systems in which classic integration methods result inefficient, among them:

- Systems with very frequent discontinuities (typical in Power Electronics).
- Large scale stiff systems (e.g. Advection–Diffusion–Reaction equation semi–discretized with the Method of Lines).

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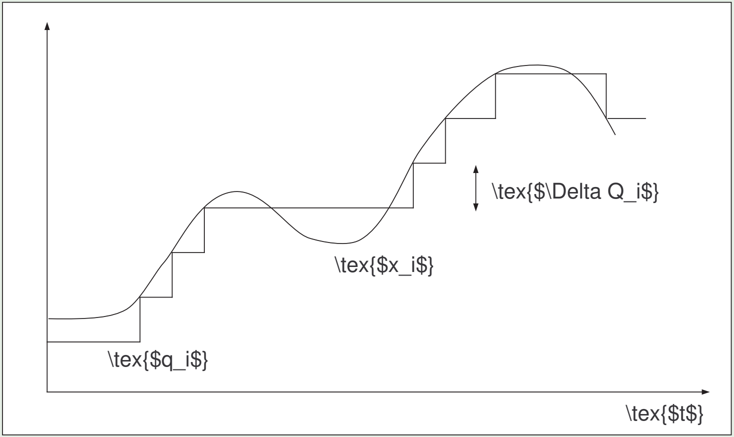
QSS1

Quantification function with hysteresis



QSS1 Method

Definition

Given the system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t)$$

the QSS1 approximation is given by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{q}(t), t)$$

where $\mathbf{q}(t)$ and $\mathbf{x}(t)$ are componentwise linked by hysteresis quantification functions.

- $\mathbf{q}(t)$ is the quantized states vector.
- Each quantification function is determined by a parameter ΔQ_i called Quantum.

QSS – Characteristics

Advantages

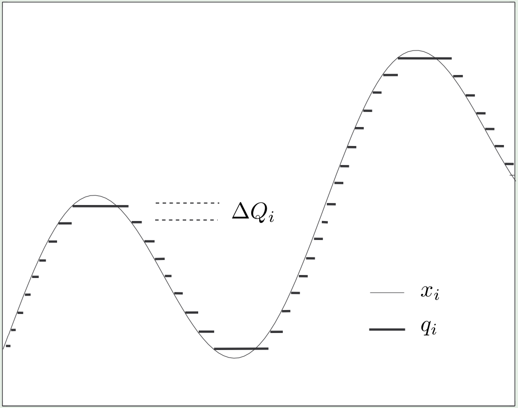
- Good stability and error bounding.
- Great advantages when simulating discontinuous systems.

Disadvantages

- Appearance of oscillations. Troubles with stiff systems.
- Number of steps grow linearly with precision.

QSS1 Method

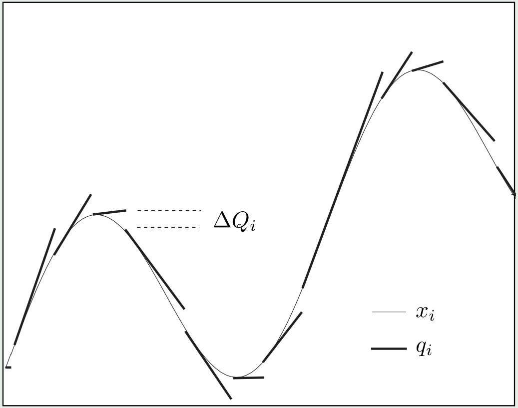
Zero order quantification



- First order method.
- Number of steps grows linearly with precision.

QSS2 Method

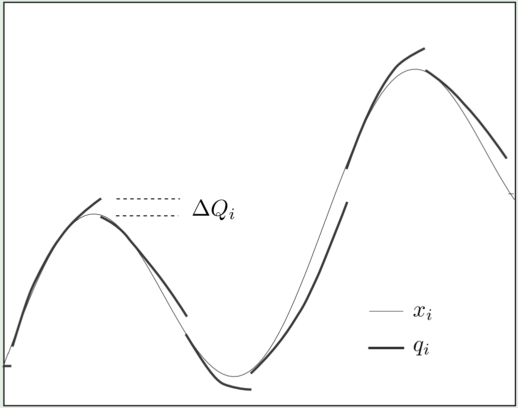
First order quantification



- Same properties and advantages as QSS1.
- Second order method.
- Number of steps grows with the square root of precision.

QSS3 Method

Second order quantification



- Same properties and advantages as QSS1.
- Third order method.
- Number of steps grows with the cube root of precision.

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Oscillations in QSS

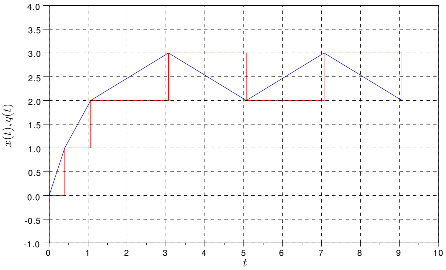
Consider

$$\dot{x}(t) = 2{,}5 - x(t)$$

and its QSS1 approximation

$$\dot{x}(t) = 2{,}5 - q(t)$$

with $\Delta Q = 1$ y $x(0) = 0$.



- Usually, QSS solutions end with oscillations around the equilibrium.
- This leads to some issues in stiff systems.

Linearly Implicit QSS Methods

Classic methods suitable for stiff systems are based on future values of the state to compute their derivatives (implicit methods).

- In QSS methods, $q_i(t)$ is always a past value of $x_i(t)$.
- The idea in LIQSS is that $q_i(t)$ takes a future state value.

Since we always know the future value in the next step ($x_i(t) \pm \Delta Q_i$), the problem results tu be explicit.

LIQSS1 Method

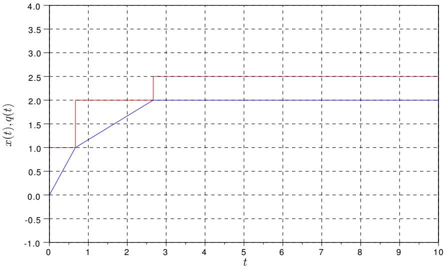
Consider the equation

$$\dot{x}(t) = 2{,}5 - x(t)$$

and its LIQSS1 approximation

$$\dot{x}(t) = 2{,}5 - q(t)$$

with $\Delta Q = 1$ y $x(0) = 0$.



In this case, final oscillations no longer exist.

LIQSS methods

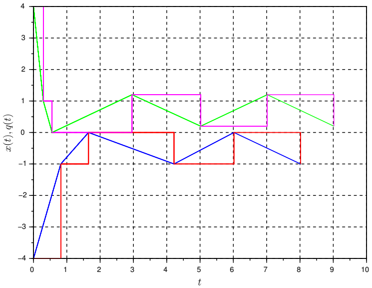
- There are LIQSS methods of order 1 to 3.
- These methods efficiently integrate stiff systems where the stiffness is due to large entries in the main diagonal of the Jacobian matrix.

Oscillations in LIQSS

Consider now

$$\dot{x}_1 = -x_1 - x_2 + 0{,}2$$
$$\dot{x}_2 = x_1 - x_2 + 1{,}2$$

with $\Delta Q = 1$, $x_1(0) = -4$ and $x_2(0) = 4$.



In this case oscillations appear due to the interaction of state variables x_1 and x_2.

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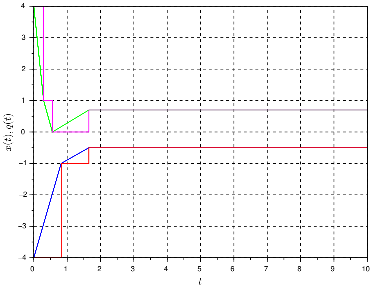
Improved LIQSS1 method

Consider now

$$\dot{x}_1 = -x_1 - x_2 + 0{,}2$$
$$\dot{x}_2 = x_1 - x_2 + 1{,}2$$

with $\Delta Q = 1$, $x_1(0) = -4$ and $x_2(0) = 4$.



Now oscillations due to the interaction of state variables x_1 and x_2 no longer exist.

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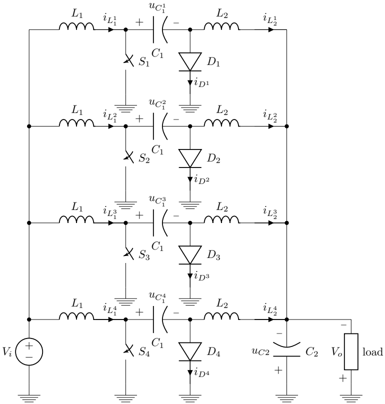
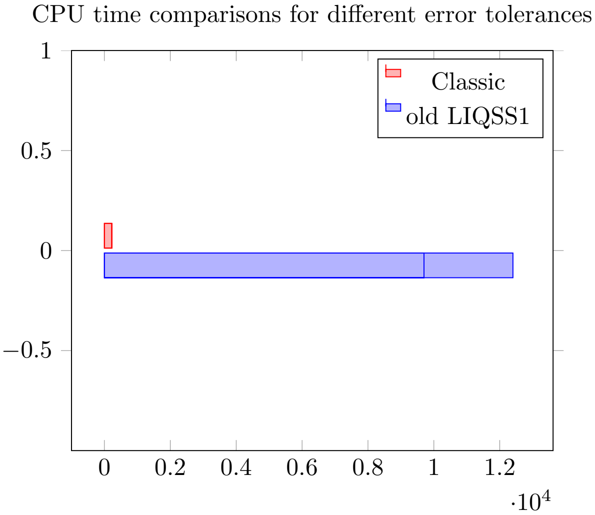
Results: Interleaved Ćuk converter



Results: Interleaved Ćuk converter

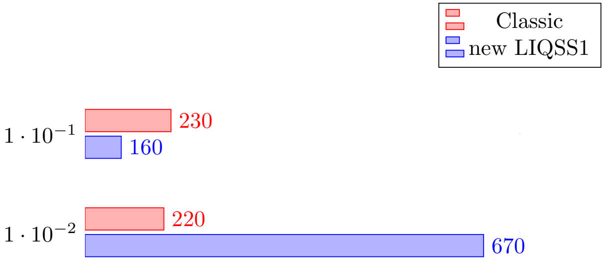


Results: Interleaved Ćuk converter

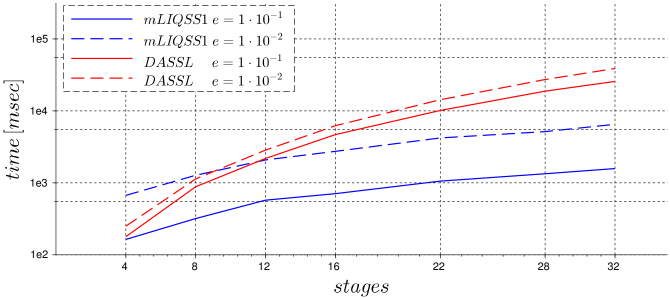
Improving LIQSS1

Franco Di Pietro

CPU time comparisons for different error tolerances



Results: Interleaved Ćuk converter



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Summary

A modification to the LIQSS1 Method was proposed.

- Allows to efficiently simulate stiff systems with more general structures than before.
- Implemented in the QSS standalone solver.
  `https://sourceforge.net/projects/qssengine/`
- It is the first theoretical step to develop higher order improved LIQSS methods.
- It is also the first approach to effectively combine QSS and classic discrete time ODE solvers.

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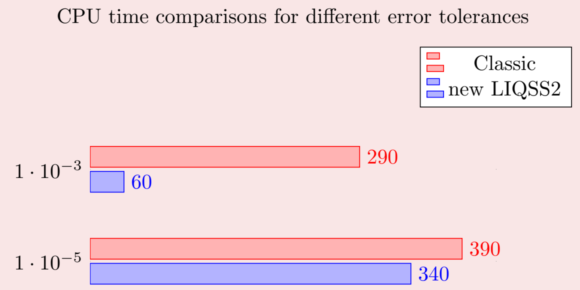
(not so) Future work

- Extending the idea for higer orders methods.



- Study these methods in a wider variety of applications.

Thank you!